

IMMC 2017 中華賽區 命題A

通過衛星遙感技術測量地球

1. 背景

地球是人類賴以生存的家園，對其形狀和各種物理現象的精細化認識一直是人們追求的目標。眾所周知，地球形狀近似橢球體，但其表面實際是一個凹凸不平的不規則體，有海洋、高山、平原、丘陵、沙漠等。精確測量地球的真實形狀和大小是衛星遙感技術的主要應用方向之一。

2. 說明

根據開普勒定律，衛星是在一個圍繞地球的橢圓軌道上運動，這個橢圓軌道可以用六個軌道要素（也叫軌道根數）描述。如圖1所示，以不轉動地心赤道坐標系XYZ為參考，六個軌道要素是：半長軸 a ，偏心率 e ，軌道傾角 i ，近地角點距 ω ，升交點赤經 Ω ，過近地點時刻 τ 。軌道上的衛星位置可以用這六個參數確定。

- (1) **軌道傾角 i** : 衛星軌道平面與地球赤道平面之間的夾角。
- (2) **半長軸 a** : 確定軌道大小的參數。圓軌道而言，就是圓的半徑；對於橢圓軌道而言，就是橢圓的半長軸。
- (3) **偏心率 e** : 確定軌道形狀的參數。 $e = 0$ 時，軌道為圓， $0 < e < 1$ 時為橢圓。
- (4) **升交點赤經 Ω** : 確定軌道平面在空間位置的參數，沿著赤道方向，春分點至升交點（衛星由南半球至北半球穿過赤道平面的點）之間的角度。
- (5) **過近地點時刻 τ** : 衛星運行中通過近地點的時刻。
- (6) **近地角點距 ω** : 沿著衛星運動的方向，從軌道升交點度量至近心點的角度，也就是交點線與近地點矢徑延長線之間的夾角。

（更詳細的衛星軌道根數介紹，可參考：https://en.wikipedia.org/wiki/Orbital_elements）

雷達高度計（Altimeter, ALT）是遙感衛星的一種重要傳感器設備，可以測量衛星與其在地球表面投影點（稱星下點）之間的距離，也就是衛星在地球表面上方的高度。衛星高度計最初的成果就是測量地球形狀及大地水準面，進而計算全球重力場。通過衛星高度計獲取的數據已廣泛地應用於海洋學、大地測量學、地球物理學、氣候學、水文地理學

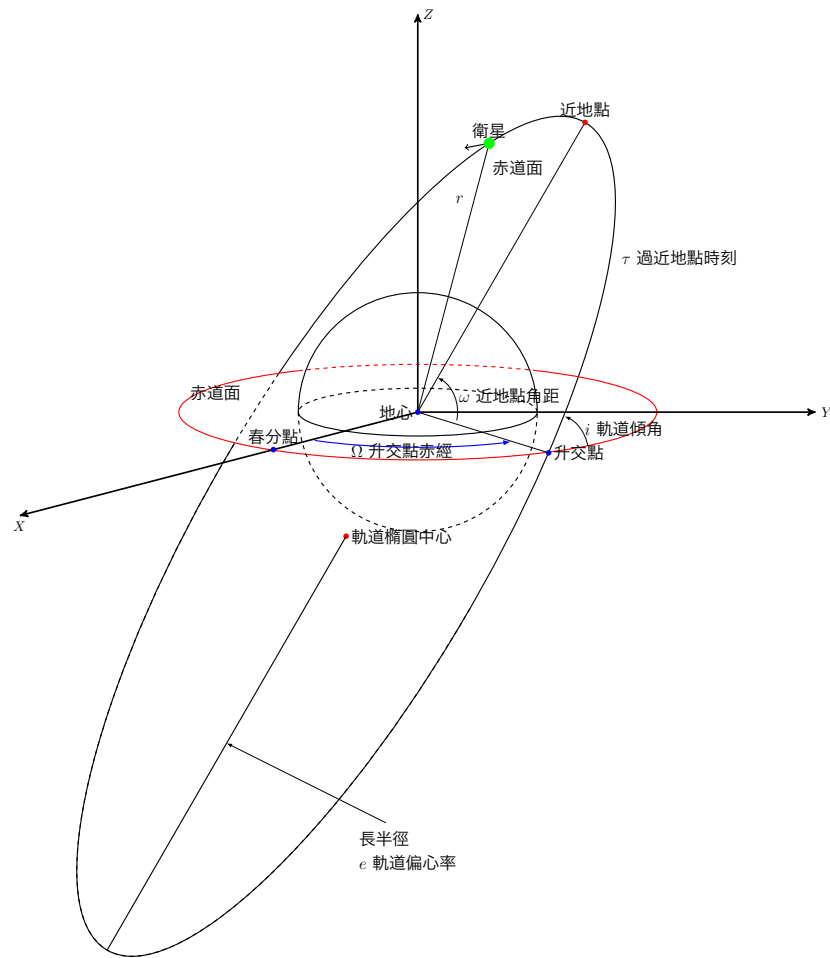


圖 1. 衛星軌道

和海洋生物學等領域的研究。尤其是在海洋學研究方面，衛星高度計為研究全球海平面、海洋環流及其隨時間的變化提供了一種強有力的工具。中國的天宮二號空間站以及多顆遙感衛星上都裝載了這種設備。

衛星雷達高度計的基本原理是通過向星下點發射無線電脈衝並測量從地球表面返回的回波的時間間隔，來測量衛星到反射表面之間的距離，如果時間間隔是 ΔT ，則高度 $H = c \cdot \Delta T / 2$ ，其中 c 是光速，如圖2所示，由於雷達高度計天線尺寸較小，其發射的無線電信號是在一個錐形空間里向下傳播的，錐角 θ 一般是 $1^\circ \sim 2^\circ$ ，照射到地

球表面的範圍近似是一個圓形（稱為足印 Footprint）。高度計的測量值實際是衛星與星下點間的距離，但簡單起見，可近似認為這個圓形區域內的地球表面離衛星的距離相同。另外，雷達高度計以很快的重複頻率發射脈衝，相鄰的足印區隔得很近，這些圓形的足印區連在一起就形成了一個帶狀的觀測區。（更詳細的衛星高度計介紹，可參考https://en.wikipedia.org/wiki/Radar_altimeter 以及<https://en.wikipedia.org/wiki/Altimeter#Satellites>

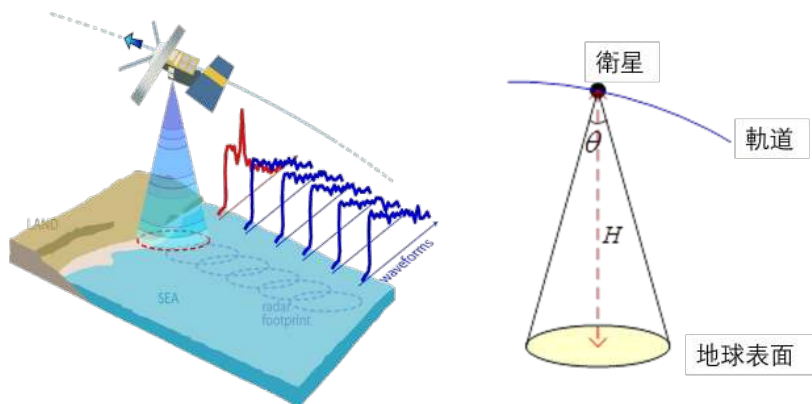


圖 2. 衛星高度計原理

3. 問題

- (1) 根據衛星軌道的簡單模型，建立用雷達高度計測量數據推算地球表面形狀的數學模型。
- (2) 對於一顆給定軌道參數及其雷達高度計錐角（或足印區大小）的衛星，建模分析最少需要多長時間才能把地球盡可能完整、均勻地測量一遍？選擇一顆在軌的遙感衛星，將其參數代入所建立的模型，計算該衛星完整測量地球所需的時間。
- (3) 如果在一顆衛星上可以安裝多個高度計，該衛星如何只利用這些高度計的測量數據確定自己的空間位置？

IMMC 2017 Region of Zhonghua Problem A

MEASUREMENTS ON EARTH VIA REMOTE SENSING TECHNIQUES

1. BACKGROUND

Our Earth is home to all human beings. We are constantly seeking new knowledge about Earth, from its physical properties to its biodiversity. Historically measuring the shape, size and the surface variations of Earth has fascinated generations of scientists. With rapid advances in technology this endeavor has become increasingly important and feasible.

It is well-known that the shape of Earth, on the macro-scale, is approximately an ellipsoid. But on the scale of our daily life its surface is in fact rough and irregular. There are oceans, high mountains, plains, hills and deserts, etc. Accurately measuring the real shape and size on Earth is one of the major application directions of remote sensing technology.

2. DESCRIPTION

Based on Kepler's Laws, the orbits of satellites are elliptic with Earth. This elliptical orbit can be described using 6 orbital elements. For example, in Figure 1, we use a non-rotational equatorial earth-centered coordinate system XYZ as reference, the 6 orbital elements are: semi-major axis α , eccentricity e , inclination i , argument of periapsis ω , longitude of the ascending node Ω , the time of perihelion passage τ . In any given time t , the orbital satellite positions can be determined by these 6 parameters.

- (1) **inclination i** : vertical tilt of the elliptical orbit of satellite with respect to the reference plane – Earth's equatorial plane.
- (2) **semi-major axis α** : this is a parameter that confirms the size of the orbit. For circular orbits, it is the radius of the circle; for elliptical orbits, this is the semi-major axis.
- (3) **eccentricity e** : this is a parameter that confirms the shape of the orbit. When $e = 0$, the orbit is a circle, when $0 < e < 1$ it is an ellipse.
- (4) **longitude of the ascending node Ω** : this is a parameter to confirm the orbital plane, as an angle measured on the Equatorial Plane from the equinox to the

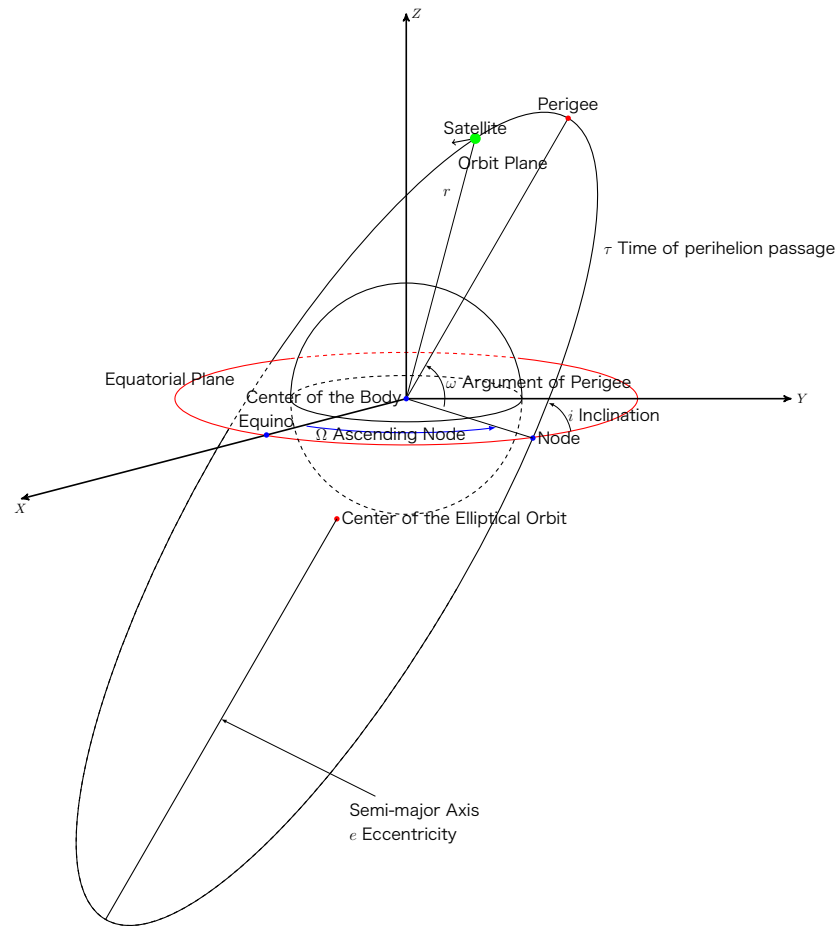


FIGURE 1. An Orbit of a Satellite

ascending node (the point where satellites will pass through on the Equatorial Plane from the Southern Hemisphere to the Northern Hemisphere)

- (5) **time of perihelion passage** τ : the time where the satellite is passing through the closest point on the central object around which it orbits.
- (6) **argument of periapsis** ω : defines the orientation of the ellipse in the orbital plane, as an angle measured from the ascending node to the periapsis (the closest point the satellite object comes to the primary object around which it orbits) and

this is also referred to as the angle between the line of ascending node and the line of perigee.

For more detailed introduction to orbital elements, please refer to:

https://en.wikipedia.org/wiki/Orbital_elements

Radar Altimeter (ALT) is an important sensor equipment for a remote sensing satellite. It can be used to measure the distance between the satellite itself and its projection point on Earth's surface (also known as the sub-satellite point (SSP)), and this is also the altitude of the satellite on Earth. The earlier results of Satellite Altimeter were to measure the shape of Earth and also the geoid, in order to calculate the global gravitational field. Through the wide coverage use of the data retrieved by the satellite altimeter, research on areas such as oceanography, geodesy, geophysics, climatology, hydrogeology and marine biology, etc., benefit enormously. Especially in the case of oceanology research, satellite altimeter provides a strong and powerful tool for research on the global oceanic surface, oceanic currents, and also the time-dependent changes associated with those data variables. Chinese Tiangong-2 space station and many more remote sensing satellites also equipped with this kind of equipment.

The basic principles of the Satellite Radar Altimeter are to transmit radio wave impulses to the sub-satellite point and measure the time lapse of the echo signal from the reflected surface on Earth, in order to measure the distance between the satellite and the reflected surface. If the time lapse is ΔT , then the height is $H = c \cdot \Delta T / 2$, where c is the speed of light. As shown in Figure 2, due to the fact that the size of the antenna is relatively small, its radio signals transmitted propagates downwards along the conical space. The angle of cone θ is usually $1^\circ \sim 2^\circ$, and the illuminated zone on Earth's surface can be approximated as a circular region (also known as Footprint). The measurement of the altimeter in fact is the distance between the satellite and the sub-satellite point. For simplicity's sake, we can take the distance between all the points on this circular region and the satellite to be the same. In addition, Radar Altimeter repeatedly emits impulses in a very rapid way, and the neighboring footprint regions are just in close proximity. The union of these circular footprint regions become a belt shaped observation zone.

For more detailed introduction on radar altimeter, please refer to

https://en.wikipedia.org/wiki/Radar_altimeter and
<https://en.wikipedia.org/wiki/Altimeter#Satellites>

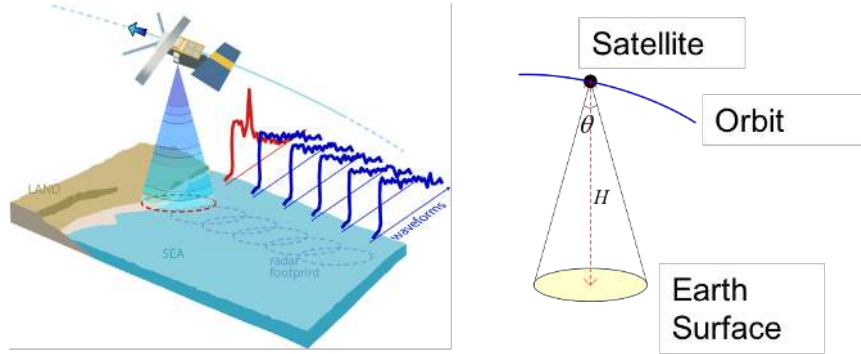


FIGURE 2. Theory Behind a Radar Altimeter

3. QUESTIONS

- (1) Based on a simple mathematical model of a satellite orbit, please construct a mathematical model using the measurement data from a radar altimeter to calculate the shape of Earth's surface.
- (2) Based on a satellite whose orbital elements and angle of cone (or size of the footprint region) are fixed, construct a mathematical model to analyze the minimum time needed to measure Earth once, as complete and even as possible. Select a remote sensing satellite in orbit, input the orbital elements into your constructed mathematical model, and calculate the time needed for that satellite to completely measure the whole Earth.
- (3) If multiple altimeters are installed on a satellite, how can that satellite to make use of measurement data by these altimeters to determine its own spatial position?